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Hydromagnetic Flow and Heat Transfer Above a Rotating Disk With Suction or Injection Considering Hall Effect

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Abstract

The steady hydromagnetic flow of a conducting fluid above a rotating disk is studied with heat transfer in the presence of uniform suction or injection without neglecting the Hall effect. The governing momentum and energy equations are solved numerically using finite differences. The results show that the inclusion of the Hall current together with the suction or injection velocity has important effects on the velocity and temperature fields.

1. Introduction

The hydrodynamic flow due to a rotating disk was studied by many researchers (von Karman, 1921; Cochran, 1934; Benton, 1966; Lee, 2003). The influence of an external uniform magnetic field on the flow due to a rotating disk was studied (El-Mistikawy and Attia, 1990; El-Mistikawy and Attia, 1991) without considering the Hall effect. The Hall effect on the rotating disk problem was considered in (Aboul-Hassan and Attia; 1997; Attia, 2002) while the ion slip effect was studied by (Attia, 2004). The effect of suction on the flow due to a rotating disk was studied in (Stuart, 1954; Ockendon, 1972). The problem of heat transfer from a rotating disk maintained at a constant temperature was first considered by (Millsaps and Pohlhausen, 1952) for a variety of Prandtl numbers in the steady state. (Sparrow and Gregg, 1960) studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. Later, many authors have studied the heat transfer near a rotating disk considering different thermal conditions (Attia, 2001; Le Palec, 1989).

In the present work the steady hydromagnetic flow with heat transfer of a viscous incompressible electrically conducting fluid due to the uniform rotation of an infinite non-conducting porous disk in an axial uniform steady magnetic field is studied in the presence of uniform suction or injection considering the Hall effect. The governing non-linear differential equations are solved numerically using finite differences to obtain the velocity and temperature distributions. The effect for the magnetic field, the Hall current and the suction or injection velocity on the velocity and temperature is reported.

2. Basic equations

The disk is assumed to be insulating and rotating in the z=0 plane about the z-axis with a uniform angular velocity ω and is maintained at a constant temperature. The fluid is assumed to be incompressible and has density ρ, kinematic viscosity ν and electrical conductivity σ. An external uniform magnetic field is applied in the z-direction and has a constant flux density $\mathbf{B}_o$. The magnetic Reynolds number is as-

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sumed to be very small, so that the induced magnetic field is negligible. The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect can not be neglected (Sutton, 1965). A uniform flow through the surface of the disk is applied. The equations of steady motion in cylindrical coordinates are given by (Attia, 2002; Sutton, 1965)

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial r}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}+\frac{\sigma B_0^2}{\rho(1+m^2)}(u-mv)+\frac{1}{\rho}\frac{\partial P}{\partial r} = V\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right), \tag{2}$$

$$u\frac{\partial u}{\partial r}+\frac{uv}{r}+w\frac{\partial u}{\partial z}+\frac{\sigma B_0^2}{\rho(1+m^2)}(v+mu) = V\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right), \tag{3}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial z}=V\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{4}$$

where u, v, and w are the velocity components in the directions of increasing r, φ, and z, p is denoting the pressure, m (=σ β B_o) is the Hall parameter which can take positive or negative values, β=1/nq is the Hall factor, n is the electron concentration per unit volume, and -q is the charge of the electron (Sutton, 1965). Due to the axial symmetry of the problem, there is no dependence on φ (von Karman, 1921). Positive values of m mean that $\mathbf{B}_o$ is upwards and the electrons of the conducting fluid gyrate in the same sense as the rotating disk. For negative values of m, $\mathbf{B}_o$ is downwards and the electrons gyrate in an opposite sense to the disk. By introducing von Karman transformations (von Karman, 1921),

$$u=rwF(\zeta),\ \ v=rwG(\zeta),\ \ w=\sqrt{wv}H(\zeta),$$
$$z=\sqrt{v/w}\zeta,\ \ p-p_\infty=-\rho vwP$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H, and P are non-dimensional functions of the modified vertical coordinate ζ, Eqs. (1)~(4) take the form

$$\frac{dH}{d\zeta}+2F=0, \tag{5}$$

$$\frac{d^2F}{d\zeta^2}-H\frac{dF}{d\zeta}-F^2+G^2-\frac{Y}{1+m^2}(F-mG)=0, \tag{6}$$

$$\frac{d^2G}{d\zeta^2}-H\frac{dG}{d\zeta}-2FG-\frac{Y}{1+m^2}(G+mF)=0, \tag{7}$$

$$\frac{d^2F}{d\zeta^2}+H\frac{dH}{d\zeta}+\frac{dP}{d\zeta}=0, \tag{8}$$

and γ is the magnetic interaction number which represents the ratio between the magnetic force to the fluid inertia force and is given by $\gamma=\sigma B_o^2/\rho\omega$ (El-Mistikawy and Attia, 1990; El-Mistikawy and Attia, 1991). The boundary conditions for the velocity field are given by

$$F(0)=0,\quad G(0)=1,\quad H(0)=\$, \tag{9a}$$
$$\zeta\to\infty,\quad F\to 0,\quad G\to 0,\quad P\to 0. \tag{9b}$$

where $ is the suction or injection parameter, which takes constant negative values for suction and constant positive values for injection. Equation (9a) indicates the no-slip conditions of viscous flow applied at the surface of the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in Eq. (9b). The above system of Eqs. (5)~(7) with the prescribed boundary conditions given by Eq. (9) are sufficient to solve for the three components of the flow velocity. Equation (8) can be used to solve for the pressure distribution if required.

Due to the difference in temperature between the wall and the ambient fluid heat transfer takes place. The energy equation, by neglecting the dissipation terms, takes the form (Sparrow and Gregg, 1960),

$$pc_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)-k\frac{\partial^2 T}{\partial z^2}=0. \tag{10}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that, by continuity considerations, the temperature equals T_w at the surface of the disk. At large distances from the disk, T tends to T_∞, where T_∞ is the temperature of the ambient fluid. In terms of the non-dimensional variable □=($T-T_\infty$)/(T_w-T_∞) and using von Karman transformations Eq. (10) takes the form,

$$\frac{1}{Pr}\frac{d^2\theta}{d\zeta^2} - H\frac{d\theta}{d\zeta} = 0, \tag{11}$$

where Pr is the Prandtl number given by , $Pr=c_p\mu/k$ The boundary conditions in terms of θ are expressed as

$$\theta(0) = 1, \quad \zeta \to \infty, \ \theta \to 0. \tag{12}$$

Since the significant velocity and temperature variations in the fluid are confined to the region adjacent to the disk, we define the thickness of these layers by certain standard measures (Sparrow and Gregg, 1960). For the tangential direction, we define a displacement thickness. Also, as a measure of the extent of the thermal layer, we may introduce a thermal thickness based on the temperature excess above the ambient. The displacement and thermal thickness are, respectively, given by

$$\delta_c = \int_0^\infty G d\zeta, \quad \delta_t = \int_0^\infty \theta d\zeta$$

The heat transfer from the disk surface to the fluid is computed by application of Fourier's law; $q=-k(\partial T/\partial z)_w$. Introducing the transformed variables, the expression for q becomes

$$q = -k(T_w - T_\infty)\sqrt{\frac{w}{v}}\frac{d\theta(0)}{d\zeta}$$

By rephrasing the heat transfer results in terms of a Nusselt number defined as, $N_u=q(\nu/\omega)^{1/2}/k(T_w-T_\infty)$ the last equation becomes

$$N_u = -\frac{d\theta(0)}{d\zeta}$$

The action of viscosity in the fluid adjacent to the disk tends to set up tangential shear stress $\bar{\tau}_\varphi$ which opposes the rotation of the disk. There is also a surface shear stress $\bar{\tau}_r$ in the radial direction. In terms of the variables of the analysis and by applying Newtonian shear stress formula (Sparrow and Gregg, 1960), the expressions of $\bar{\tau}_\varphi$ and $\bar{\tau}_r$ are respectively given as

$$\frac{\bar{\tau}_\phi}{pr\sqrt{vw^3}} = \tau_\phi = \frac{dG(0)}{d\zeta}, \quad \frac{\bar{\tau}_r}{pr\sqrt{vw^3}} = \tau_r = \frac{dG(0)}{d\zeta}$$

The system of non-linear differential equations (5)~(7) and (11) is solved under the conditions given by Eqs. (9) and (12) using the Crank-Nicolson implicit method (Ames, 1977) to obtain the velocity and temperature distributions. The resulting system of equations has to be solved in the infinite domain $0<\zeta<\infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for $\zeta_\infty=10$ and step size $\Delta\zeta=0.04$ which are found adequate for the ranges of the parameters studied here. Larger finite distance or smaller step size do not show any significant change in the results. Convergence of the scheme is assumed when any one of F, G, H, θ, dF/dζ, dG/dζ, and dθ/dζ for the last two approximations differs from unity by less than 10^{-6} for all values of ζ in $0<\zeta<10$. It should be pointed out that the results obtained herein reduce to those reported by (Aboul-Hassan and Attia, 1997; Attia, 2002) when $\$=0$. Also, the steady state solutions reported by (Attia, 2001) are reproduced by setting $\$=0$ and m=0 in the present results.

3. Results and discussion

Figures 1a and b presents the variation of the axial velocity at infinity H_∞ with the Hall parameter m and the suction parameter \$ and for γ=0.5 and 1, respectively. It is clear that increasing the parameter \$ increases H_∞ for all values of m and γ. The maximum value of H_∞ occurs at negative and small values of m for all values of γ. Increasing γ reduces the axial flow towards the disk, i.e. increases H_∞. For small values of γ, and all values of m, as shown in Fig. 1a, the axial flow at infinity is towards the disk for higher values of γ, while as indicated in Fig. 1b, H_∞ reverses direction in the case of negative and small values of m for $\$\geq0$.

Figures 2a and b presents the variation of the displacement thickness δ_c with the Hall parameter m and the suction parameter \$ and for γ=0.5 and 1, respectively. Increasing \$ increases δ_c for all values of m and γ. The maximum value of δ_c occurs at small negative values of m while a minimum value for δ_c

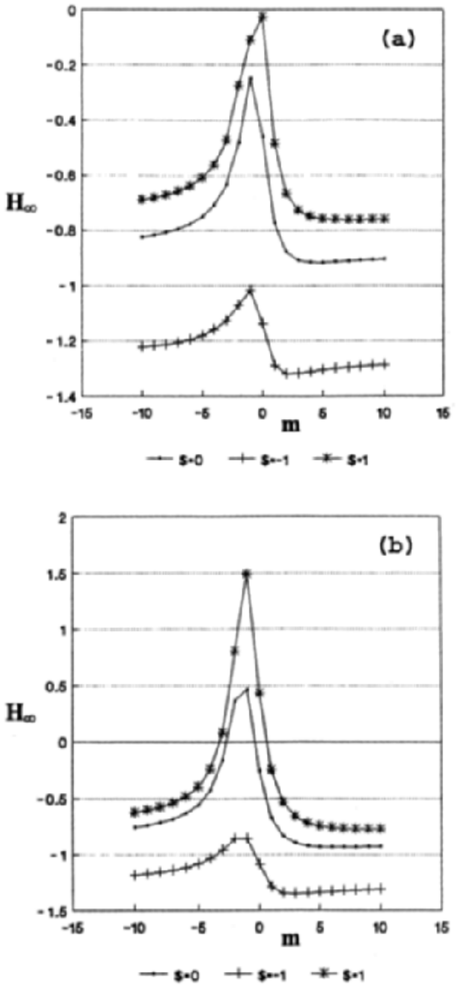

Fig. 1. Variation of H_∞ with m for various values of $: (a) γ=0.5, and (b) γ=1.

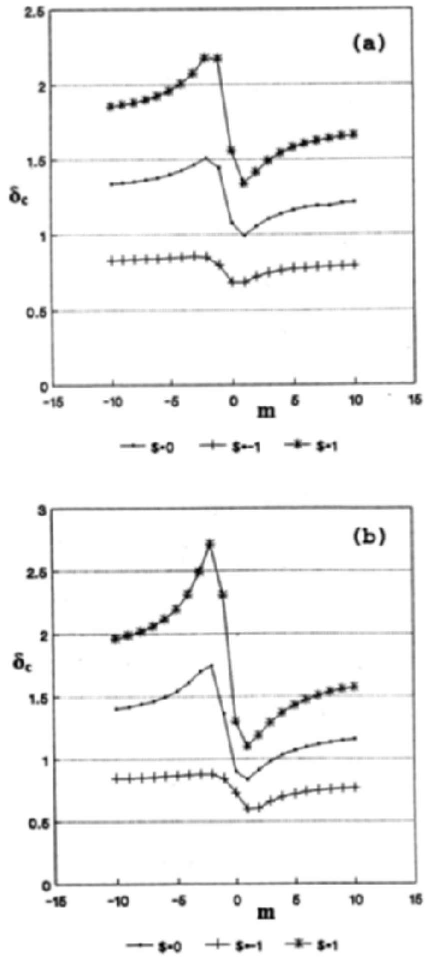

Fig. 2. Variation of δ_c with m for various values of $: (a) γ=0.5, and (b) γ=1.

occurs at small positive values of m for all values of γ. It is clear that the effect of γ on δ_c depends on m and $. In general, increasing γ increases δ_c for negative m, but it decreases δ_c for positive values of m. The effect of γ on δ_c becomes more apparent for higher values of $.

Figures 3a and b presents the variation of the thermal thickness δ_t with the Hall parameter m and the suction parameter $ for Pr=10 and for γ=0.5 and 1, respectively. Increasing $ increases δ_t for all values of m and γ. The effect of m on δ_t can be neglected entirely in the suction case ($=-1). The maximum value of δ_t occurs at small negative values of m. The effect of γ on δ_t depends on m and $. In the suction case $<0, increasing γ has no significant effect on δ_t. When $=0, increasing γ increases δ_t for negative m but it decreases δ_t for positive m. In the injection case, $>0, increasing γ increases δ_t for all positive or negative values of m.

Tables 1a-c present the effect of the parameters m and $, respectively, on the axial flow at infinity H_∞, the radial shear stress τ_r, and the tangential shear stress τ_φ and for γ=1. As shown in Table 1a, Increasing $ decreases the axial flow towards the disk for all values of m. For positive or negative large values of m, the axial flow at infinity keeps its direction towards the disk. For the case m=0 or small positive values of m, the axial flow reverses direction for the injection case. It is of interest to find also that, in the case of small negative values of m, the axial flow reverses direction in the injection case and even in the case of zero suction or injection. Table 1b in-

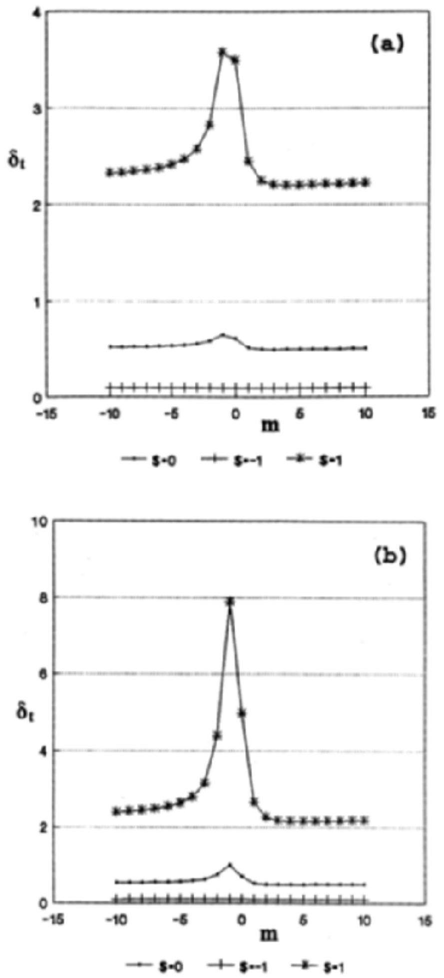

Fig. 3. Variation of δ_t with m for various values of $: (a) γ=0.5, and (b) γ=1.

dicates that for the suction case, $<0, increasing $ increases τ_r but for the injection case, $>0, increasing $ decreases τ_r for positive or large negative values of m. However, for small negative values of m, increasing the injection velocity increases τ_r, but increasing $ more decreases τ_r. For positive values of m and all values of $ except for large suction velocities ($=-2), increasing m increases τ_r. For the case of large suction velocity and large positive values of m, increasing m decreases τ_r. For small and negative values of m, increasing the magnitude of m decreases τ_r, on the other hand, for large negative values of m, increasing the magnitude of m increases τ_r. As clear from Table 1c, increasing $ decreases the magnitude of τ_φ for all values of m. For positive or negative values of m, increasing the magnitude of m

Table 1. The effect of the parameters m and $ on: (a) H_∞, (b) τ_r, and (c) τ_φ.

(a) H_∞	M=-5	m=-0.5	m=0	m=0.5	m=5
$=-2	-2.0139	-1.9821	-2.0319	-2.0824	-2.0933
$=-1	-1.0863	-0.9526	-1.0899	-1.2144	-1.3383
$=0	-0.5561	0.0903	-0.2534	-0.5098	-0.9314
$=1	-0.3962	1.0109	0.4325	0.0227	-0.7455
$=2	-0.3303	1.7346	1.0162	0.4463	-0.6407

(b) τ_r	m=-5	m=-0.5	m=0	m=0.5	m=5
$=-2	0.1508	0.0502	0.1889	0.3362	0.3235
$=-1	0.2649	0.0844	0.2512	0.4301	0.4765
$=0	0.4070	0.1452	0.3093	0.4953	0.5792
$=1	0.4021	0.1831	0.3217	0.4895	0.5532
$=2	0.3249	0.1745	0.2915	0.4324	0.4587

(c) τ_φ	m=-5	m=-0.5	m=0	m=0.5	m=5
$=-2	-2.0334	-2.3412	-2.4305	-2.3924	-2.0818
$=-1	-1.1225	-1.5278	-1.6566	-1.6339	-1.2638
$=0	-0.5446	-0.9132	-1.0691	-1.0626	-0.7088
$=1	-0.2632	-0.5578	-0.6912	-0.681	-0.3698
$=2	-0.1209	-0.3684	-0.4668	-0.4453	-0.1803

decreases the magnitude of τ_φ for all values of $.

Tables 2a-c present the effect of the parameters m and $, respectively, on the displacement thickness δ_c, the thermal thickness δ_t, and the Nusselt number N_u. The results are estimated for γ=1 and Pr=10. Table 2a shows increasing $ increases δ_c for all values of m. For negative values of m, increasing the magnitude of m increases δ_c for all values of $. For the case of injection and small suction velocity and for small positive values of m, increasing m decreases δ_c. However, for large positive values of m, increasing m increases δ_c. Also it is found that in the case of large suction velocity and positive values of m, increasing m increases δ_c. As shown in Table 2b, increasing $ increases δ_t for all values of m and its effect is more pronounced for higher values of $. For positive values of m, increasing m decreases δ_t. For small negative values of m, increasing the magnitude of m increases δ_t, but for large negative values of m, increasing the magnitude of m decreases δ_t. Table 2c indicates that increasing $ decreases N_u for all values of m by blanketing the surface with fluid whose temperature is closed to T_w. For small values of m, increasing m

Table 2. The effect of the parameters m and $ on: (a) δ_c, (b) δ_t, and (c) N_u.

(a) δ_c	m=-5	m=-0.5	m=0	m=0.5	m=5
$=-2	0.4907	0.4267	0.4101	0.4126	0.4737
$=-1	0.8743	0.6530	0.5970	0.5862	0.7348
$=0	1.5439	1.0846	0.8989	0.8304	1.0675
$=1	2.1953	1.7176	1.3038	1.1278	1.4282
$=2	2.8672	2.4267	1.7749	1.4741	1.8523

(b) δ_t	m=-5	m=-0.5	m=0	m=0.5	m=5
$=-2	0.0498	0.0499	0.0498	0.0498	0.0498
$=-1	0.0988	0.0995	0.0989	0.0983	0.0980
$=0	0.5684	0.9162	0.7003	0.5682	0.4949
$=1	2.6288	7.2658	4.9860	3.3776	2.1573
$=2	4.3454	9.1751	8.9685	6.6310	3.5789

(c) N_u	m=-5	m=-0.5	m=0	m=0.5	m=5
$=-2	-20.009	-20.005	-20.0102	-20.016	-20.0158
$=-1	-10.0375	-10.0095	-10.033	-10.0587	-10.068
$=0	-1.0421	-0.7117	-0.8811	-1.0579	-1.1881
$=1	-0.0034	-0.0034	-0.0034	-0.0034	-0.0034
$=2	-0.0071	-0.0071	-0.0071	-0.0071	-0.0071

increases N_u, however, for large positive or negative values of m, increasing the magnitude of m increases N_u. The effect of m on N_u is more pronounced for smaller values of $.

4. Conclusion

The steady MHD flow of a conducting fluid with heat transfer due to an infinite rotating porous disk are studied in the presence of a uniform suction or injection through the surface of the disk considering the Hall effect. The magnetic field, the Hall effect and the suction or injection velocity have interesting effects on the velocity and temperature fields. It is shown that the sign of the Hall parameter is important and the effect of the Hall parameter on the velocity and temperature fields is more pronounced for higher values of the magnetic field and the injection velocity. It is of interest to see the reversal of the direction of the axial velocity component for some values of the Hall parameter and the suction or injection velocity. It is also shown that the effect of the magnetic field on the displacement thickness and the thermal thickness depends on the Hall parameter and the suction or injection velocity.

It should be mentioned that for all the figures and tables given above The case $=0 corresponds to the results presented by (Attia, 2001) while the case m=0 corresponds to those obtained by (Aboul-Hassan and Attia, 1997; Attia, 2002).

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